MET 4501 Spring 2025: In-Class Quiz 2 Name: SOLUTION

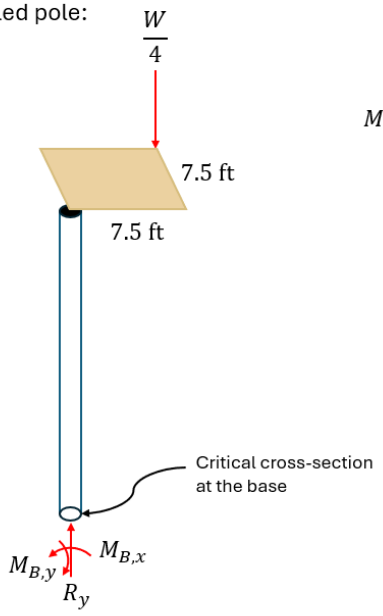
INSTRUCTIONS:

This quiz is open-book and open-note. Please answer all questions and show all of your work.

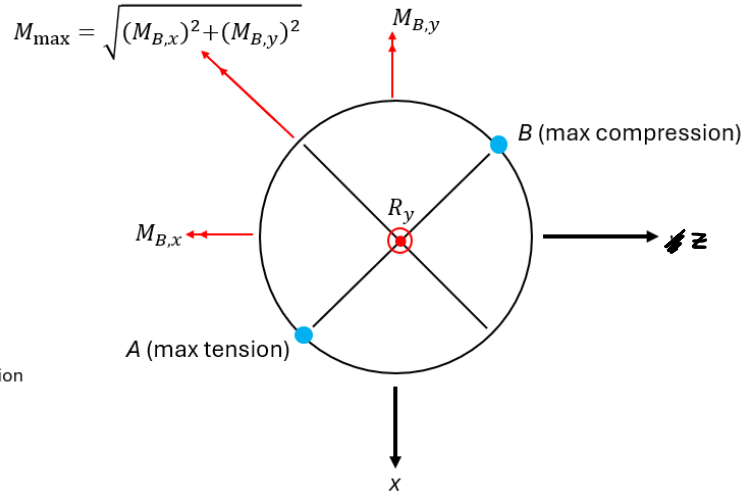
GIVEN:

Recall the shelter and pole from In-Class Quiz 1, illustrated below with the forces and moments acting at the cross-section of the pole at the base. Recall that point A was a critical point on the cross section. 2 in DIAMETER

FBD of circled pole:

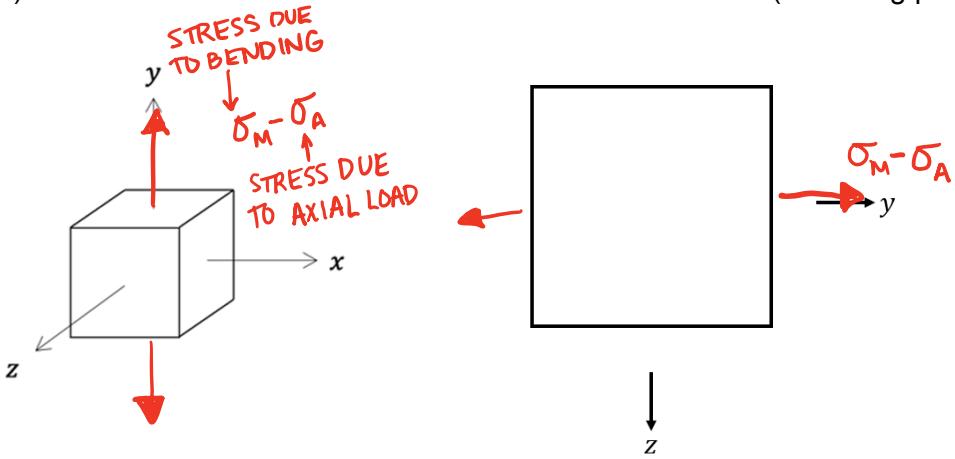


Cross-section at base:



FIND:

1) (10 points) Draw the stresses on the element below in 3D and in 2D (assuming plane stress).



THIS IS A PLANE STRESS STATE WITH $\sigma_1 = \sigma_M - \sigma_A$, $\sigma_2 = 0$, $\sigma_3 = 0$

2) (10 points) Given that $M_{\max} = 132.6\ \text{ft}\cdot\text{lbf}$ and $R_y = 12.5\ \text{lbf}$, calculate the numerical values of each stress acting on the stress element.

$$I = \frac{\pi r^4}{4} = \frac{\pi(1\ \text{in})^4}{4} = 0.785\ \text{in}^4$$

$$\sigma_A = \frac{R_y}{A} = \frac{12.5\ \text{lbf}}{\pi(1\ \text{in})^2}$$

$$\sigma_A = 3.98\ \text{psi}$$

$$\sigma_M = \frac{M_{max}\, y}{I} = \frac{(132.6\ \text{ft}\cdot\text{lbf})\left(\frac{12\ \text{in}}{\text{ft}}\right)(1\ \text{in})}{0.785\ \text{in}^4} = 2027\ \text{psi}$$

$$\sigma_1 = \sigma_{total} = 2027 - 3.98\ \text{psi} = 2023\ \text{psi}$$

3) Assume that the pole is a ductile hot-rolled steel bar with a minimum yield strength in tension and compression of 50 ksi. Determine the factor of safety at point A using:
   - (20 points) The distortion-energy (DE) theory
   - (20 points) The maximum-shear-stress (MSS) theory

DE:
$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} = \sqrt{\frac{\sigma_1^2+\sigma_1^2}{2}} = \sigma_1$$

$$n = \frac{S_y}{\sigma'} = \frac{50\ \text{ksi}}{2023\ \text{psi}} = \frac{50000\ \text{psi}}{2023\ \text{psi}} = 24.7$$

MSS:
$$n = \frac{S_y}{\sigma_1-\sigma_3} = \frac{S_y}{\sigma_1}$$

$$n = \frac{50000}{2023} = 24.7$$

4) Assume that the pole is a (brittle) gray cast iron bar with a minimum ultimate strength in tension of 30 ksi and a minimum ultimate strength in compression of 130 ksi. Determine the factor of safety at point A using:
   - (20 points) The brittle Coulomb-Mohr (BCM) theory
   - (20 points) The modified-Mohr (MM) theory

BCM: $\sigma_A \geq \sigma_B \geq 0$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{S_{ut}}{\sigma_1} = \frac{30\ \text{ksi}}{2023\ \text{psi}} = \frac{30000\ \text{psi}}{2023\ \text{psi}}$$

$$n = 14.8$$

MM: $\sigma_A \geq \sigma_B \geq 0$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{S_{ut}}{\sigma_1} = \frac{30000\ \text{psi}}{2023\ \text{psi}}$$

$$n = 14.8$$

BONUS: (10 points) Which material is the best choice for this application (the ductile HR steel or the brittle gray cast iron)? Explain your reasoning.

ALL OTHER CONSIDERATIONS BEING EQUAL, THE HR DUCTILE STEEL IS THE BEST MATERIAL CHOICE BECAUSE IT RESULTS IN THE HIGHEST SAFETY FACTOR.